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Full Length Article

Transport network downsizing based on optimal sub-network

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ABSTRACT

Transportation networks are sized to efficiently achieve some set of service objectives. Under particular circumstances, such as the COVID-19 pandemic, the demand for transportation can significantly change, both qualitatively and quantitatively, resulting in an over-capacitated and less efficient network. In this paper, we address this issue by proposing a framework for resizing the network to efficiently cope with the new demand. The framework includes a model to determine an optimal transportation sub-network that guarantees the following: (i) the minimal access time from any node of the *urban network* to the new sub-network has *not excessively increased* compared to that of the original *transportation network*; (ii) the delay induced on any itinerary by the removal of nodes from the original transportation network has not *excessively increased*; and (iii) the number of removed nodes from the transportation network is within a preset known factor. A solution is optimal if it induces a minimal global delay. We modelled this problem as a Mixed Integer Linear Program and applied it to the public bus transportation network of Lyon, France, in a case study. In order to respond to operational issues, the framework also includes a decision tool that helps the network planners to decide which bus lines to close and which ones to leave open according to specific trade-off preferences. The results on real data in Lyon show that the optimal sub-network from the MILP model can be used to feed the decision tool, leading to operational scenarios for network planners.

1. Introduction

In transportation networks, one of the big issues is matching supply and demand. When supply is too low in relation to demand, some users will see their journeys negatively impacted: delays, increased travel time, or even the impossibility of making certain journeys. If supply is too high in relation to demand, this will lead to costs for transport operators, jeopardizing the sustainability of the system.

During the COVID-19 pandemic, demand was impacted in two ways. On the one hand, during containment and curfews, overall mobility decreased sharply, reaching almost zero during containment. During these periods, transport networks adapted in an emergency, limiting their services to a minimum, severely handicapping certain areas. On the other hand, even during periods of partial or total recovery, demand for public transport did not reach its pre-crisis level, resulting in revenue shortfalls for operators. While the former changes in demand can be assumed to be one-time, the latter can be long-lasting. The solutions found in periods of containment are not applicable in the long term for reasons of mobility equity. However, we need to resize transport networks to enable transport operators to provide a financially sustainable service.

This article focuses on public transportation bus networks, and aims to provide a decision tool to network managers when dramatic alterations of the demand arise. In particular, we assume that the demand has decreased and that such new demand is *known exactly* in the form of an Origin-Destination (OD) matrix. However, our method can be adapted in order to match an increased demand situation. The proposed decision tool provides several alternative scenarios of reconfiguration of the transit offer. Each scenario contains a set of existing bus lines to close and the corresponding delay profile induced by the line's closure. The network managers will have then to choose between the different provided scenarios according to the trade-off between operational costs and quality of service they prefer. The scenarios are based on an optimization model defined with respect to the bus stops of the existing transit network. The model finds an optimal sub-network that guarantees that (1) the minimal access time from any node of the urban network to the new sub-network has *not excessively increased* compared to that of the original transportation network; (2) the delay caused on any itinerary, by the exclusion of nodes of the transportation network, is not *excessively increased*; and (3) the number of excluded bus stops from the transportation network is within a preset known factor. To do so, we assume that the delay is due to the difference of access time in the network. The choice of being interested only in the bus network is motivated by the

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dynamic aspect we want: our decision tool is to be flexible in order to match even an uncertain demand. Thus, the choice of the lines to close can change regularly (every day, every week …) which is possible only with light infrastructures such as bus stops. We apply the generic framework from the optimization problem to the decision tool in a case study in Lyon's urban area, France, in which we use a real dataset.

The paper is organized as follows: in Section 2 we give elements of literature that are useful to understand the context in which this article is placed. In Section 3, we present the global framework leading to generic construction of the decision tool for network managers. We first introduce the optimization model to find an optimal sub-network before describing the construction of the set of scenarios. In Section 4, we present the case study with the bus public transportation network of Lyon, France. In this case study, we detail the real dataset we use before applying the generic model on it and analyse the results. Finally, we conclude the paper with a description of ongoing and future works.

2. Literature review

Public transportation networks in large urban areas carry a very large number of passengers everyday. The number of network users, which constitutes the demand for transport in a given area, is usually estimated from surveys or diverse sources of data (e.g., passengers' count sensors, mobile phones, etc.). Based on such estimation, a public transportation vehicles fleet, the supply, is deployed to satisfy such demand. The supply is defined to match the demand as much as possible. In some circumstances, however, the demand may fluctuate dramatically due to several causes, such as vacations periods, heavy pollution (Nkurunziza et al. (2012); Deschaintres et al. (2019); Manley et al. (2018)).

The match between supply and demand can be performed and adjusted over time. Knowing the demand, one can build or rebuild the network topology accordingly. Such problems go under the name of network design, for which Newell (1979) did a seminal work. Guihaire and Hao (2008) present a global review of network design approaches, by also suggesting solutions for adjusting timetables of transit lines and bus frequency to meet a modified travel demand. LeBlanc (1988) defines transport network design as the problem of finding the optimal frequencies of transit lines. Lee and Vuchic (2005) also consider the possibility that the transit demand could change over time. In this paper, the authors assume that the total demand (cars and transit network) is fixed, but the proportion of demand for transit network can vary. More recently Cipriani et al. (2012) have proposed a case study of network design for the city of Rome, with multi-modal properties and complex road network topology. Lo et al. (2013) consider network design under demand uncertainty for ferries. Ukkusuri and Patil (2009) focus instead on multi-stage network design in order to accommodate demand fluctuations and demand uncertainty.

Even though these approaches can be able to deal with slight demand variations, in this work we focus on dramatic demand fluctuations, and, specifically, on drastic reductions of the demand. To the best of our knowledge, classical network design techniques do not cope well with such demand variations, once the network is set. In such situations, one can modify transit timetabling and transit schedules to meet the modified demand. For instance, D'Acierno et al. (2014) propose network re-planning strategies to deal with budget cuts. Even though the demand is not assumed to decrease in this work, the proposed approaches can be easily adapted to make the supply match a decreasing demand. Similarly, Kang et al. (2015) focus on rescheduling last train transit lines in case of delays during the day, while Gkiotsalitis and Stathopoulos (2016) are interested in rescheduling time tables during leisure activities in vacation periods. All these techniques are very useful when small demand variations arise. However, when the variation of the travel demand is too large, the topology of the network has to change in order to limit the operational costs. The recent confinements due to COVID-19 are periods during which such a variation took place CRMIDE (2021).

New mobility services, such as complete or hybrid on-demand transports, can also be considered as a means to accommodate demand variations. Brake et al. (2004), and more recently Davison et al. (2014) focus, in a survey, on the development of Demand Responsive Transports (DRT) in UK. An evaluation of the efficiency of DRT across Europe is presented by Mageean and Nelson (2003). The authors present the key issues that could influence the introduction of DRT in daily mobility. However in our case, we would like to use the preexisting network in order to get benefit from all the currently available supply.

Finally, some authors focused on on-time solutions when the demand changes frequently. Huang et al. (2020) imagine a highly flexible situation wherein demand fluctuations can be continuously accounted for by adjusting the path of the buses based on passengers' varying demand. Dobler et al. (2012) used within-day route and timetabling replanning when an exceptional event arises. These events could imply a drastic growth of the demand (sport events, concerts, evacuation …), or a decrease of it. Gu et al. (2020) study the impact of perturbations on the transportation network under three main concepts: reliability, vulnerability, and resilience.

All the previous methods and approaches are not sufficiently efficient in the case of a large decrease of the demand. For instance, during the current global pandemic linked to COVID-19, the demand has changed both qualitatively and quantitatively (Di Renzo et al. (2020); Tirachini and Cats (2020); Liu et al. (2020); CRMIDE (2021); Chowdhury et al. (2021)). On the one hand, during lockdown, the users of the network adopted completely different habits: some no longer used it (due to e.g., practicing total or partial smart work from their home) and some would use individual transportation modes rather than collective ones to achieve their work location. When modifications of the demand occur, the network design techniques are too rigid to accommodate further possible variations, while transit timetabling techniques cannot only adapt to minor changes of the demand.

Regarding network resizing, one of the big issues is the increase in access time. Hamacher et al. (2001) develop a simple model to minimize the door-to-door time. Schöbel (2005) defines a very interesting mathematical model with two objective functions, one minimizing the access time and the second adding bus stops to compensate for the loss of access time by increasing the travel time. Demetsky et al. (1982) and Murray et al. (1998) treat the problem of vertex overlap in a graph. This problem is very relevant in the case where one wants to resize a transport network while limiting the increase in access time. Laporte et al. (2002) are interested in the positioning of stations when creating a new line, which could also be another approach to our problem by reasoning directly on new lines.

Notations

Notation	Description
PTN	Public Transportation Network
V	Vertices of PTN
$n_t = \|V\|$	Total number of nodes in PTN
E_t	Edges of PTN
$m_t = \|E_t\|$	Total number of edges in PTN
c	Cost function (traveling time) over E_t
$PC(v_1, v_2)$	Traveling time of the shortest path between $v_1 \in V$ and $v_2 \in V$
UN	Urban Network
$V \cup U$	Vertices of UN
$n_u = \|U\|$	Total number of nodes in U
E_u	Edges of UN
$m_u = \|E_u\|$	Total number of edges in UN
$d(u, v)$	Access time between $u \in U$ and $v \in V$
$d_{acc}(u)$	Minimum access time of $u \in U$ to PTN
$acc(u)$	Closest bus stop from $u \in U$
$OD(u_1, u_2)$	Demand from $u_1 \in U$ to $u_1 \in U$
p_{elim}	Minimum ratio of bus node to exclude
α	Acceptable increase factor of the delay
$k(u)$	Acceptable increase factor of the access time of $u \in U$
$V_{sol} \subseteq V$	Subset of V which represents a solution
$d_{acc}^{V_{sol}}(u)$	Minimum access time of $u \in U$ in V_{sol}
$Opt_{V_{sol}}(u_1, u_2)$	Optimal travel time from $u_1 \in U$ to $u_2 \in U$ for a solution V_{sol}
$TWT_{V_{sol}}$	Total weighted traveling time of V_{sol}

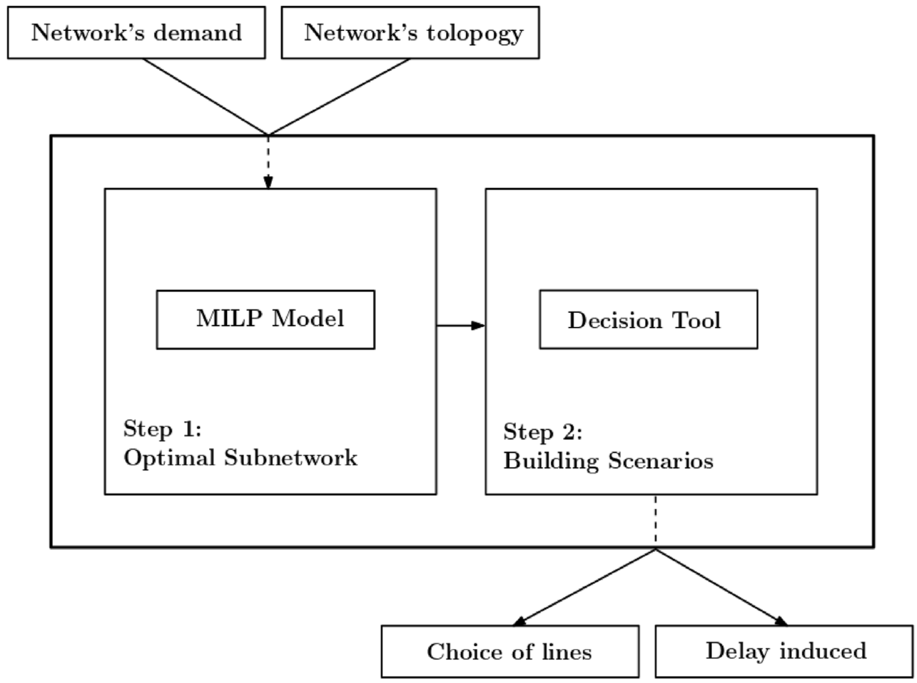


Fig. 1. Generic framework.

3. Generic framework

In the following, we present the general framework for the identification of the optimal sub-network for a reduced network demand, which constitutes the basis for the definition of a decision tool producing network reconfiguration scenarios for network managers. As a main input to our framework, we consider a bus public transportation network with known topology, i.e., a set of bus stops and a set of lines, the latter corresponding to sequences of bus stops. We also assume, as a second input to our framework, the knowledge of the public transport demand, i.e., the number of users wanting to travel from a given origin O to a certain destination D by relying on the bus network above. Both origins and destinations are defined in terms of urban zones corresponding to a given geographic partitioning of the urban public transport network.

The framework relies on the topology of the network and on the demand in order to compute an optimal sub-network in terms of bus stops. This model is based on linear programming. Specifically, as the input demand is considered to drop with respect to a known reference typical situation, the framework produces, as a first output, a new network that is smaller, in the number of bus nodes, than the reference one. In a second step, the optimal sub-network produced as the first output is leveraged to build multiple, *more realistic*, public bus network reconfiguration scenarios, each consisting in: (*i*) a set of bus lines to close and (*ii*) a distribution of the delay induced by the reduction of the number of lines. The generic framework is presented in Fig. 1, while the two main building blocks of the framework are formally described in next two subsections.

3.1. Step 1: Discovery of the optimal sub-network

The first block of the proposed framework aims at solving the problem of discovering the optimal sub-graph from the input public transport network in the case of a drastically reduced transit demand (the problem is denoted as Optimal Sub-Network Problem (OSNP) in the following). Concerning the modelling choices of the considered problem, we assume the existence of two kinds of networks atop the urban area of interest: (*i*) an *urban network* (*UN*), which corresponds to a geographical segmentation of the considered urban area, and (*ii*) a *public transportation network* (*PTN*) which represents the current existing bus network covering the aforementioned urban area. Moreover, we assume that the shortest-path travel time between any pair of nodes of PTN is known as well as the access time between any node of the urban network UN and any node of the transportation network PTN. As previously discussed, we also assume that we know the modified (reduced) demand (which has been the case during the COVID-19 pandemic). A solution to our problem is a sub-network of the original transportation network guaranteeing that: (*i*) the minimal access time from any node of the urban network to the new network is not *too large* compared to the original transportation network; (*ii*) for any itinerary, the delay caused by the deletion of nodes of the transportation network is not *too large*; and (*iii*) the number of nodes of the transportation network has been reduced at least by a known factor. A solution is optimal if it induces a minimal global delay. We describe more formally the problem in the next section.

3.1.1. Definition of the problem

Let $PTN = (V, E_t)$ be an undirected graph representing the *Public Transportation Network* of a given city, with $|V| = n_t$ nodes and $|E_t| = m_t$ edges. Each node $v \in V$ represents a bus stop of the public transport network, while each edge $(v_i, v_j) \in E_t$ corresponds to any bus route connecting stops v_i and v_j, i.e., at least a bus line exists that serves the two nodes. Let $c : E_t \mapsto \mathbb{R}_+$ be a cost function over the edges of PTN representing the *travel time*. Let $PC \in \mathbb{R}_+^{n_t \times n_t}$ be the known matrix of the shortest path travel time between each pair of bus stops. The shortest travel time between the generic pair of nodes $v_i, v_j \in V$ is denoted as $PC(v_i, v_j)$ in the following. Note that each $PC(v_i, v_j)$ is assumed to represent an *average travel time* and is considered to be static, i.e., not influenced by traffic conditions or by the removal of the bus stops themselves.[1] Fig. 2 reports a simple example of the PTN for six bus stops and eight connecting bus routes.

Let $UN = (U \cup V, E_u)$ be a complete bipartite graph representing the *Urban Network*, i.e., the known segmentation of the city in traffic analysis

[1] In this paper, we do not focus on short-term decisions but rather on medium-to-long term decisions by a transport operator to handle significant variations of the transit demand. For that reason, we do not consider traffic dynamics but rather the average travel time information to determine the shortest paths.

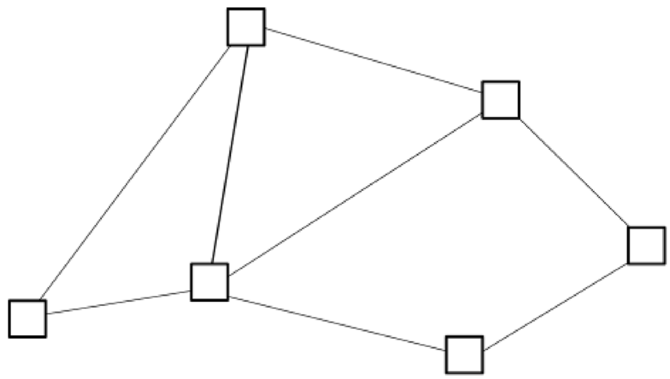


Fig. 2. PTN for six bus stops.

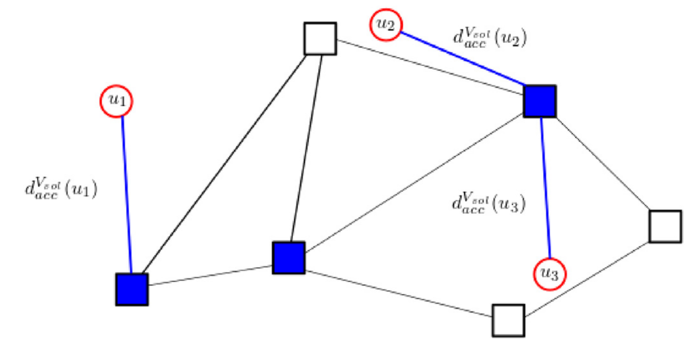
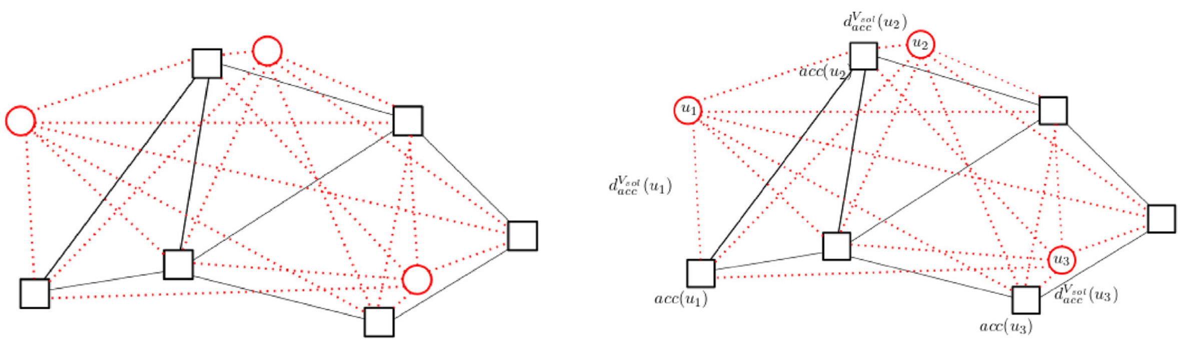


Fig. 4. In blue: V_{sol} and the corresponding $d_{acc}^{V_{sol}}$.



(a) PTN (in black) and UN (in red) **(b)** PTN, UN and the corresponding d_{acc} and acc

Fig. 3. PTN and UN for a simple example bus network.

zones. The nodes U are the $n_u = |U|$ centroids of the identified urban areas, which are associated to the origins and destinations of the known perturbed demand. E_u represents the complete set of links between each urban area and each bus stop. As UN is complete, the users can theoretically reach any bus station from any origin zone of the city, e.g., by walk. To have a more realistic model, let $d : U \times V \mapsto \mathbb{R}_+$ be the *access time* from an urban area u to a bus stop v, with $d(u, v)$ representing the average time that the users have to walk to reach bus stop $v \in V$ from the urban area $u \in U$. For any $u \in U$, we thus denote by $d_{acc}(u) = \min_{v \in V} d(u, v)$ the minimum access time for u and by $acc(u) = argmin_{v \in V} d(u, v)$ the bus stop this minimum refers to. Let $OD \in \mathbb{Z}_+^{n_u \times n_u}$ the known origin/destination bus transit demand: $OD(u_i, u_j)$ is the number of users who wish to go from urban area $u_i \in U$ to $u_j \in U$. Figs. 3a and 3b provide the graphical representations of PTN, UN and the corresponding d_{acc} and acc.

An instance of our problem is a tuple $I = (PTN, c, PC, UN, d, OD, p_{elim}, \alpha, k)$, where PTN, c, PC, UN, d and OD are defined as above. Moreover, we assume the knowledge of the following parameters:

- $p_{elim} \in [0, 1]$ is the minimum percentage of the bus stop that have to be excluded;
- $1 < \alpha \in \mathbb{R}_+$ is the maximum acceptable increase factor of the delay associated to the new network we want to design for each pair of origins and destinations;
- $k \in \mathbb{R}_+^{n_u}$, where $k(u)$ is the maximum acceptable increase factor for the access time of $u \in U$.

A solution to our problem is a subset V_{sol} of V. For a solution $V_{sol} \subseteq V$ and an origin/destination pair $(u_1, u_2) \in U \times U$, the optimal travel time from u_1 to u_2 is defined as

$$Opt_{V_{sol}}(u_1, u_2) = \min_{(v_1,v_2) \in V_{sol}^2} d(u_1, v_1) + PC(v_1, v_2) + d(u_2, v_2) \quad (1)$$

We also define the total weighted traveling time of a solution V_{sol} as

$$TWT_{V_{sol}} = \sum_{(u_1,u_2) \in U^2} OD(u_1, u_2) Opt_{V_{sol}}(u_1, u_2) \quad (2)$$

In particular, in the original network PTN, the optimal traveling time between u_1 and u_2 is

$$Opt_V(u_1, u_2) = \min_{(v_1,v_2) \in V^2} d(u_1, v_1) + PC(v_1, v_2) + d(u_2, v_2) \quad (3)$$

and the total weighted traveling time of PTN is $TWT_V = \sum_{(u_1,u_2) \in U^2} OD(u_1, u_2) Opt_V(u_1, u_2)$.

Finally, we define the minimum access time for all $u \in U$ for a solution V_{sol} as $d_{acc}^{V_{sol}}(u) = \min_{v \in V_{sol}} d(u, v)$. In particular, $d_{acc} = d_{acc}^{V}$.

As $|V| \geq |V_{sol}|$, we have $TWT_V \leq TWT_{V_{sol}}$ and, as a consequence, the total weighted delay induced by the choice of solution V_{sol} (i.e., the deletion of all the nodes in $V \setminus V_{sol}$) is $TWT_{V_{sol}} - TWT_V \geq 0$. Fig. 4 represents an example of solution (nodes from V_{sol} are represented as blue squares) and the corresponding $d_{acc}^{V_{sol}}$.

Real-world instances of OSNP contain hundreds of bus stops, and several thousands of origin/destination pairs. Thus, computing $Opt_{V_{sol}}(u_1, u_2)$ for all the possible choice of V_{sol} is not feasible and solving this problem on such instances is not possible without making simplifying assumptions. We will now introduce the main assumptions of this article which made possible the computational resolution of our problem on real-world instances.

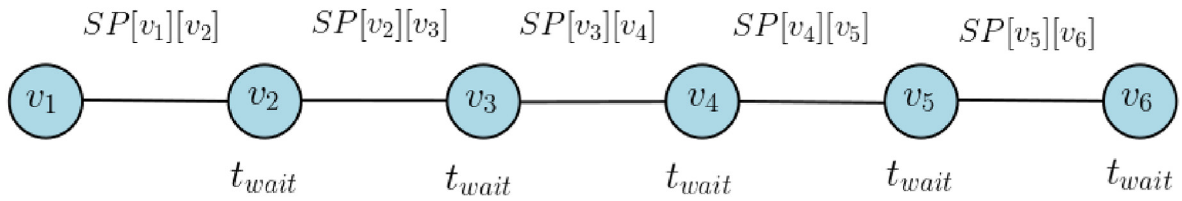


Fig. 5. Example of a bus line before deletion.

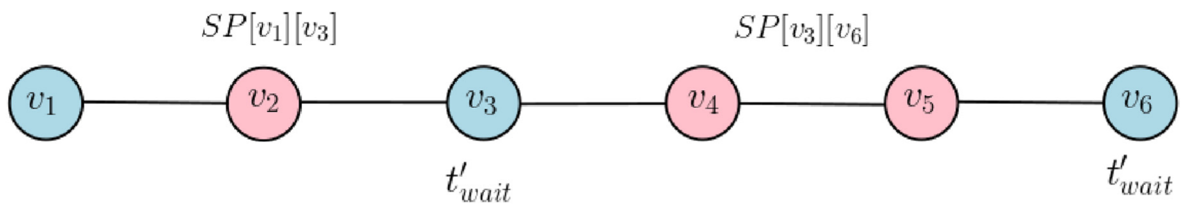


Fig. 6. Example of a bus line after deletion.

Assumption. We assume that the delay due to the deletion of bus stops is caused by the difference of access time, and not by the difference of the shortest path in the bus network. Thus, we have

$$Opt_{V_{sol}}(u_1,u_2) = \min_{(v_1,v_2)\in V^2} d(u_1,v_1) + PC(acc(u_1),acc(u_2)) + d(u_2,v_2) \tag{4}$$

Note that with this assumption, we have

$$Opt_{V_{sol}}(u_1,u_2) = d_{acc}^{V_{sol}}(u_1) + d_{acc}^{V_{sol}}(u_2) + PC(acc(u_1),acc(u_2)) \tag{5}$$

Remark. This assumption can be considered quite strong, as, by it, we omit the difference of travel time between the original network and the choice induced by V_{sol}. We also assume implicitly that the faster way to go from u_1 to u_2 is to pass through the nearest bus stops to u_1 and u_2. This is not true in general even if this simplification induces a minimum walking time. With this assumption, we do not have $TWT_V \leq TWT_{V_{sol}}$ in general. We will analyse the consequences of such an assumption in the next sections.

However, this assumption can be justified as follows. Consider the initial bus network. When a bus serves the stops that are on the route, the overall travel time can be divided into two types: the driving time on the one hand, which is the time the bus will actually move; and the stopping time at each bus stop. Let us assume that the stopping time at each stop is constant in the initial network.

Now consider that we have, thanks to our model, removed a number of bus stops. On this degraded line, the travel time will be smaller, since the shortest path between two successive stops will not necessarily pass through the stops that have been closed. On the other hand, we assume that the total travel time is the same. To ensure this, we can assume that the stopping time at each bus stop is greater in the case of a degraded line. This assumption seems acceptable to us. Indeed, as some stops have been removed, users will access the bus stops potentially from further away, and there will be more people at each stop. The increased dwell time at each bus stop will allow the driver to better manage the flow of riders entering and exiting the bus. Moreover, we can bound precisely the stopping time in the degraded line. Let us take an example of a line of l bus stops where some bus stops have been deleted (see Figs. 5 and 6). Let t_{wait} be the stopping time at each bus stop before deletion, and t'_{wait} be the stopping time after the deletion.

Let TT be the travel time of the bus before deletion and TT' after. We have, if $x_i = 1$, the bus stop i remains, and 0 otherwise. We also remind that here l is the length of the bus line.

$$TT = \sum_{i=1}^{l-1} (t_{wait} + SP[v_i][v_{i+1}]) \tag{6}$$

$$TT' \leq \left(\sum_{i=1}^{l-1} x_i\right) \times t'_{wait} + \sum_{i=1}^{l-1} x_i SP[v_i][v_{i+1}] \tag{7}$$

Then, as we assume that $TT = TT'$, we can compute the minimum value for t'_{wait}, considering that the driving time remains the same.

$$t'_{wait} \geq \frac{(l-1)t_{wait} + \sum_{i=1}^{l-1} SP[v_i][v_{i+1}] - \sum_{i=1}^{l-1} x_i SP[v_i][v_{i+1}]}{\sum_{i=1}^{l-1} x_i} \tag{8}$$

Based on the aforementioned considerations, we want our solution V_{sol} to satisfy the following properties:

A) the access time increases from $u \in U$ induced by the choice of V_{sol} must not increase more than by a factor $k(u)$
B) the delay induced by the choice of V_{sol} must not increase more than by a factor α
C) the percentage of deletion of V_{sol} has to be at least p_{elim}

More formally, we want V_{sol} to satisfy:

A) $\forall u \in U,\ d_{acc}^{V_{sol}}(u) \leq k(u) \times d_{acc}(u)$
B) $\forall (u_1,u_2) \in U^2,\ Opt_{V_{sol}}(u_1,u_2) \leq \alpha \times Opt_V(u_1,u_2)$
C) $|V_{sol}| \leq (1 - p_{elim})|V|$

A solution V_{sol} to our problem is said to be *feasible* if it satisfies the constraints above. Let us call $\mathcal{V}_{\mathcal{F}}$ the set of all feasible solutions. As there is a finite number of bus stops, we know that $|\mathcal{V}_{\mathcal{F}}|$ is finite too. Our goal is to find a solution that minimizes the total weighted traveling time. So V^*_{sol} is said to be *optimal* if it verifies $TWT_{V^*_{sol}} = \min_{V\in\mathcal{V}_{\mathcal{F}}} TWT_V$. The Optimal Sub-Network Problem (OSNP) is the problem consisting in finding such a solution.

Mathematical programming and specifically linear programming appear as a natural choice for the formulation of the OSNP, as the objective is to minimize the total weighted traveling time under specific (linear) constraints.

3.1.2. Mixed-integer linear programming formulation

Let us define, for all $v \in V$ and all $k \in \mathbb{R}_+^{n_u}$, $D_u^k = \{v \in V | d(u,$

$v) \leq k(u)d_{acc}(u)\}$. We also define M as an upper bound on the $d(u, v)$.

Let us consider the mathematical program (P):

$$
\begin{array}{llllll}
min & \sum_{(u_1,u_2)\in U} OD(u_1,u_2)(d^x_{acc}(u_1)+d^x_{acc}(u_2)) & & & & \\
s.t. & \sum_{v\in D^k_u} x(v) & \geq & 1 & \forall u \in U & (1)\\
 & d^x_{acc}(u) & \leq & d(u,v)+(1-x(v))M & \forall u\in U, \forall v \in D^k_u & (2)\\
 & d^x_{acc}(u_1)+d^x_{acc}(u_2)+PC(acc(u_1),acc(u_2)) & \leq & \alpha(d_{acc}(u_1)+d_{acc}(u_2)+PC(acc(u_1),acc(u_2))) & \forall (u_1,u_2)\in U^2 & (3)\\
 & \sum_{v\in V} x(v) & \leq & (1-p_{elim})n_t & & (4)\\
 & x & \in & \mathbb{Z}_+{}^{n_t} & & \\
 & d^x_{acc} & \in & \mathbb{R}_+{}^{n_u} & &
\end{array}
$$

Before proving formally that this MILP answers our problem, let us explain the corresponding variables and constraints. This MILP contains two kinds of variables. The x variables are such that $x(v) = 1$ if we keep v in the subnetwork and 0 otherwise. The $d^x_{acc}(u)$ are meant to be the access time of u in the subnetwork. Indeed, as M is big enough, and thank to the minimization, we have $d^x_{acc}(u) = \min_{v\in D^k_u} d(u,v) + (1 - x(v))M$. So $d^x_{acc}(u)$ is the minimum of the $d(u, v)$ if v is kept in the subnetwork.

Constraint (1) ensures that at least one bus stop is accessible in reasonable time from any $u \in U$. Constraint (2) makes sure that $d^x_{acc}(u)$ is the new access time from any $u \in Y$. Constraint (3) ensures that the travel time of any trip does not increase more that by a factor α. Finally, Constraint (4) ensures that the deletion ratio of the bus stops is at least p_{elim}. Let us prove formally that (P) models our problem.

Let us call $\mathcal{P}_\mathcal{F}$ the set of feasible solutions of (P).

Theorem. The two following propositions hold:

i) There is a one-to-one correspondence between the feasible solutions of (P) and the feasible solutions of the OSNP;
ii) There is a one-to-one correspondence between the optimal solutions of (P) and the optimal solutions of the OSNP.

Proof. i) Let $V_{sol} \in \mathcal{V}_\mathcal{F}$ be a feasible solution of the OSNP. We define $x_{V_{sol}} \in \mathbb{Z}_+{}^{n_t}$ as a 0/1 vector describing whether or not a bus stop is selected as part of the solution. More formally, for all $v \in V$:

$$x_{V_{sol}}(v) = \begin{cases} 1 & \text{if } v \in V_{sol} \\ 0 & \text{otherwise} \end{cases}$$

Note that the decision variables d^x_{acc} are entirely set by the definition of x with Constraint (2) of (P). Let us prove that $(x_{V_{sol}}, d^{x_{V_{sol}}}_{acc})$ is a feasible solution of (P).

As $V_{sol} \in \mathcal{V}_\mathcal{F}$, we know by hypothesis that:

A) $\forall u \in U$, $d^{V_{sol}}_{acc}(u) \leq k(u) \times d_{acc}(u)$
B) $\forall (u_1, u_2) \in U^2$, $Opt_{V_{sol}}(u_1,u_2) \leq \alpha \times Opt_V(u_1,u_2)$
C) $|V_{sol}| \leq (1 - p_{elim})|V|$

From A), we know that for all $u \in U$, $\min_{v\in V_{sol}} d(u,v) \leq k(u)d_{acc}(u)$. Let $v^* \in V_{sol}$ such that $v^* = argmin_{v\in V_{sol}} d(u,v)$. We have $d(u, v^*) \leq k(u)d_{acc}(u)$. Thus we have $v^* \in V_{sol} \cap D^k_u$, which implies that $V_{sol} \cap D^k_u$ is not an empty set and Constraint (1) is satisfied.

From B) and Assumption (5), and by noticing that $d^{x_{V_{sol}}}_{acc} = d^{V_{sol}}_{acc}$ we have

$$d^{V_{sol}}_{acc}(u_1) + d^{V_{sol}}_{acc}(u_2) + PC(acc(u_1), acc(u_2)) \leq \alpha(d^V_{acc}(u_1) + d^V_{acc}(u_2) + PC(acc(u_1), acc(u_2)))$$

Thus Constraint (3) is satisfied.

From C) we know that the number of 0 in $x_{V_{sol}}$ must be more than $p_{elim}n_t$, so Constraint (4) is satisfied. Thus, $(x_{V_{sol}}, d^{x_{V_{sol}}}_{acc})$ satisfies all the constraints of (P), so it is a feasible solution for (P).

Reciprocally, let $x \in \mathcal{P}_\mathcal{F}$ be a feasible solution of (P). We define $V_{sol} \subseteq V$ as $V_{sol} = \{v \in V | x(v) = 1\}$. From Assumption (1), we know that for all $u \in U$, $\{v \in V_{sol} | d(u, v) \leq k(u)d_{acc}(u)\} \neq \emptyset$, and *a fortiori* $d^{V_{sol}}_{acc}(u) \leq k(u) \times d_{acc}(u)$, and V_{sol} satisfies A).

As $d^x_{acc} = d^{V_{sol}}_{acc}$ by the definition of V_{sol}, the constraints (2), (3) and assumption (5) induce that V_{sol} satisfies B).

Just like before, Constraint (4) and the definition of V_{sol} induce that $|V_{sol}| \leq (1 - p_{elim})n_t$, C) is satisfied and ($i$) is proved.

ii) As there is a one-to-one correspondence between the feasible solutions of (P) and the feasible solutions of the OSNP, we just have to prove that the objective functions of both problems are the same. Let x be a feasible solution of (P) and the corresponding $V_{sol} \in \mathcal{V}_\mathcal{F}$ defined just like before. We have

$$
\begin{aligned}
&\min_{x\in\mathcal{P}\_\{\mathcal{F}\}} \sum_{(u_1,u_2)\in U^2} OD(u_1,u_2)(d^x_{acc}(u_1)+d^x_{acc}(u_2)) \\
&= \min_{x\in\mathcal{P}\_\{\mathcal{F}\}} \sum_{(u_1,u_2)\in U^2} OD(u_1,u_2)(d^x_{acc}(u_1)+d^x_{acc}(u_2)+PC(acc(u_1),acc(u_2))) \\
&= \min_{V\in\mathcal{V}\_\{\mathcal{F}\}} \sum_{(u_1,u_2)\in U^2} OD(u_1,u_2)(d^V_{acc}(u_1)+d^V_{acc}(u_2)+PC(acc(u_1),acc(u_2))) \\
&= \min_{V\in\mathcal{V}\_\{\mathcal{F}\}} \sum_{(u_1,u_2)\in U^2} OD(u_1,u_2)Opt_{V(u_1,u_2)} \\
&= \min_{V\in\mathcal{V}\_\{\mathcal{F}\}} TWT_V
\end{aligned}
$$

Thus, if $V \in \mathcal{V}_\mathcal{F}$ minimizes TWT_V, the corresponding $x \in \mathcal{P}_\mathbf{F}$ minimizes

$$\sum_{(u_1,u_2)\in U^2} OD(u_1,u_2)\big(d^x_{acc}(u_1)+d^x_{acc}(u_2)\big)$$

and *ii*) is proved, which proves the theorem.

We can now find an optimal solution of the OSNP by finding an optimal solution of (P). Such a solution can be found using standard MILP solving algorithms. We will now describe how to use the optimal subnetwork found to identify a set of scenarios that will provide a decision tool for network planners.

3.2. Decision tool

In this section, we propose a decision tool to provide transit planners with recommendation scenarios for adapting their network to a reduced demand. We assume that the OSNP has been solved, and an optimal

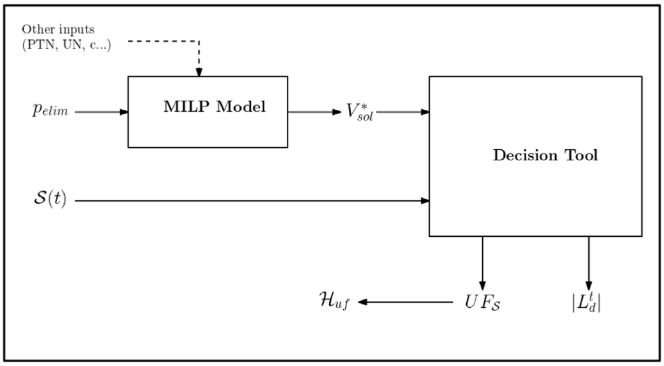


Fig. 7. Schema of the construction of a scenario $\mathcal{S}$.

solution V_{sol} has been computed as its output. This solution provides important information on the bus stops a transit service provider could consider to reduce the size of the transit network: bus stops that are not in V_{sol} appear less relevant than those in V_{sol} to the performance and accessibility of the network with respect to the observed demand. However, removing bus stops that are not included in V_{sol} could result in a poor way of optimizing costs from an operational point of view: it is in fact the bus lines and associated trip schedules that induce most of the operation costs and that should be thus considered for closure in response to variations of the demand (Litman, 2015). Based on such considerations, one should consider a way to convert the optimal sub-network (corresponding to the optimal solution V^*_{sol} of our OSNP, and composed of a reduced number of bus stops) into a choice of bus lines to retain or close. It is likely that the bus stops included in V^*_{sol} are not evenly distributed over the different bus lines of the transit network, and, consequently, some lines are more worthy than others to be retained to efficiently satisfy the demand reduction. In particular, one could think to retain only the lines for which the percentage of stops included in V^*_{sol} is high, while closing those for which such percentage is low.[2] Based on such idea, we introduce a threshold t representing the percentage of the nodes of a line included in the optimal solution V^*_{sol} below which the line will be proposed as a candidate for closure. The proposed decision tool analyzes multiple scenarios (i.e., different choices for threshold t) with respect to V^*_{sol}, and for each analyzed scenario outputs the set of lines with a percentage of remaining open stops lower than t that should be potentially closed. The tool also produces, as output, the delay induced by the deletion of the corresponding lines for each scenario.

More formally, let $l = (s_1, .., s_{|l|} \in V)$ represent a line, i.e., a sequence of bus stop nodes from V. For an optimal solution V^*_{sol}, we define the percentage of remaining open stops of l as $p^{V^*_{sol}}_{ros}(l) = \frac{|V^*_{sol} \cap l|}{|l|}$. Let L be the set of all lines. We define L^t_k (k for keep) and L^t_d (d for deleted) a partition of L such that $L^t_k = \{l \in L | p^{V^*_{sol}}_{ros}(l) \geq t\}$ and $L^t_d = \{l \in L | p^{V^*_{sol}}_{ros}(l) < t\}$. Our approach consists in evaluating, for several values of t, the sets L^t_k and L^t_d. If t is close to 0, then L^t_k will be close to L, and the percentage of lines deleted will be small, maybe too small for the network planner. If t is close to 1, then L^t_d will be close to L and the network efficiency will be widely degraded. The tool we propose allows analyzing different scenarios and choose the one(s) that provide an acceptable trade-off between cost and efficiency for the network planner.

To build a scenario $\mathcal{S}$ (a choice of t), we begin by choosing a value for p_{elim} and we compute an optimal solution V^*_{sol} thanks to the model described in Section 3.1.2. Then, we choose $t \in [0, 1]$ and compute the corresponding L^t_k and L^t_d. We arise with a new solution (which is not feasible in general regarding (P)) $V_{\mathcal{S}} = V^*_{sol} \setminus L^t_d$, which contains the stops in V^*_{sol}, minus the stops of the lines in L^t_d. On the one hand, we have deleted exactly $|L^t_d|$ lines from L, which would help the network planner to manage the transit offer with respect to the modified demand. On the other hand, since the solution is no longer feasible in general, there will be some $u \in U$ for which the access time Constraint (1) from problem formulation (P) is violated. Let $UF_{\mathcal{S}}$ be the set of such $u \in U$. Let us call $uf(u) = k(u) \times d_{acc}(u) - d^{V_{\mathcal{S}}}_{acc}(u)$ the difference between the maximal access time from $u \in U$ allowed by Constraint (1) of (P) and the actual access time in $V_{\mathcal{S}}$: $d^{V_{\mathcal{S}}}_{acc}(u)$ is the minimal access time to a bus stop in $V_{\mathcal{S}}$: $d^{V_{\mathcal{S}}}_{acc}(u) = \min_{v \in V_{\mathcal{S}}} d(u, v)$. Then, $UF_{\mathcal{S}} = \{u \in U | uf(u) < 0\}$. For a given scenario $\mathcal{S}$, our tool is therefore also able to provide the histogram $\mathcal{H}_{uf}$ of the $uf(u)$. This information could be useful to the network planner to choose the best trade-off scenario among the different analyzed ones. We describe schematically our decision tool for the evaluation of line-removal scenarios $\mathcal{S}$ in Fig. 7.

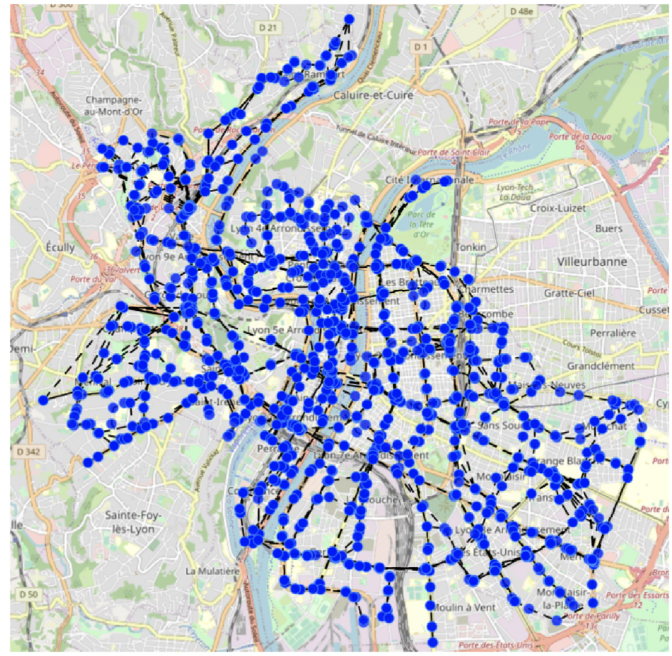


Fig. 8. Map of Lyon's urban area and the 1144 bus stops in it.

In the previous sections we have described the generic model to build a decision tool for network planners. It provides them a set of scenarios that includes different trade-offs between the number of remaining lines in the network and the induced delay distribution. Now we will apply the model on the public transportation bus network of Lyon, with real data. In the next sections, we will describe the dataset before presenting the numerical results and an analysis of them. We want these sections to serve as a proof of concept of the generic model.

4. Case study

4.1. Dataset

In this section we describe the dataset we used for the real study case. As shown in Fig. 1, we need to know the topology of the network and the demand of it as inputs. For the topology of the *PTN* network, we use the data of the transportation network of Lyon (TCL, 2021). The database contains 1144 geographic positions of all the bus stops. The edges are built from Open Street Map (2019), which is a collaborative project which maps the topology of the roads all around the world. A map of the corresponding bus stops is represented Fig. 8. The cost c of the edges are the length of these roads. Consequently, the matrix PC of the shortest

[2] In an optimal situation, these percentages are either 0 or 1 and the decision of which lines to keep is trivial.

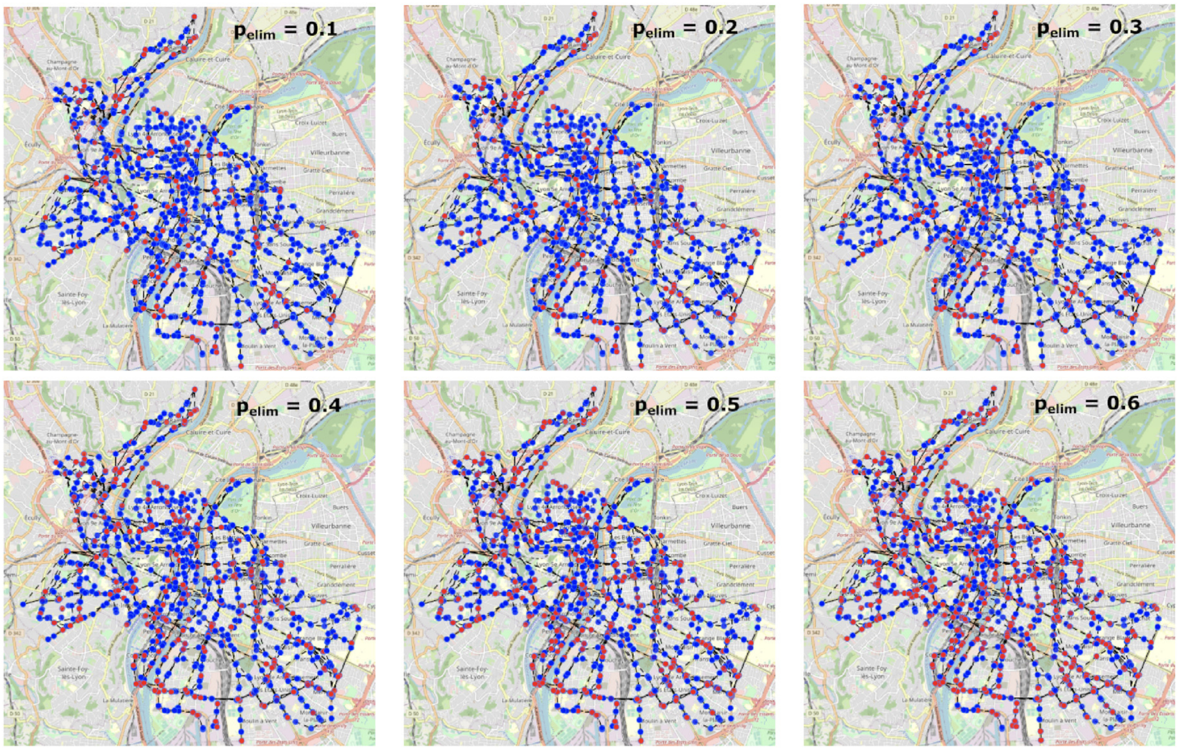


Fig. 9. Map of remaining open stops in blue, and deleted stops in red for different values of p_{elim}.

Table 1
Number of deleted bus stops for p_{elim} between 0.1 and 0.6.

p_{elim}	$\lceil p_{elim} n_t \rceil$	# deleted bus stops
0.1	115	266
0.2	229	295
0.3	344	344
0.4	458	458
0.5	572	572
0.6	687	687

paths between all pairs of nodes in *PTN* is computed from these real distances. For the topology of the *UN*, the city was segmented into 1464 urban zones based on the French IRIS zones segmentation, defined by the French institute of statistics and economic studies (INSEE, 2021). We further divided the IRIS zones into smaller ones in order to have better spatial granularity, and selected the nodes of the *UN* as the centroids of the defined sub-zones. The access time *d* between the nodes of *UN* and those of *PTN* is defined as the haversine distance between the corresponding geographic points.

For the demand, the *OD* has been built iteratively with the LICIT traffic simulation open-source tool SymuVia (2021), combined with

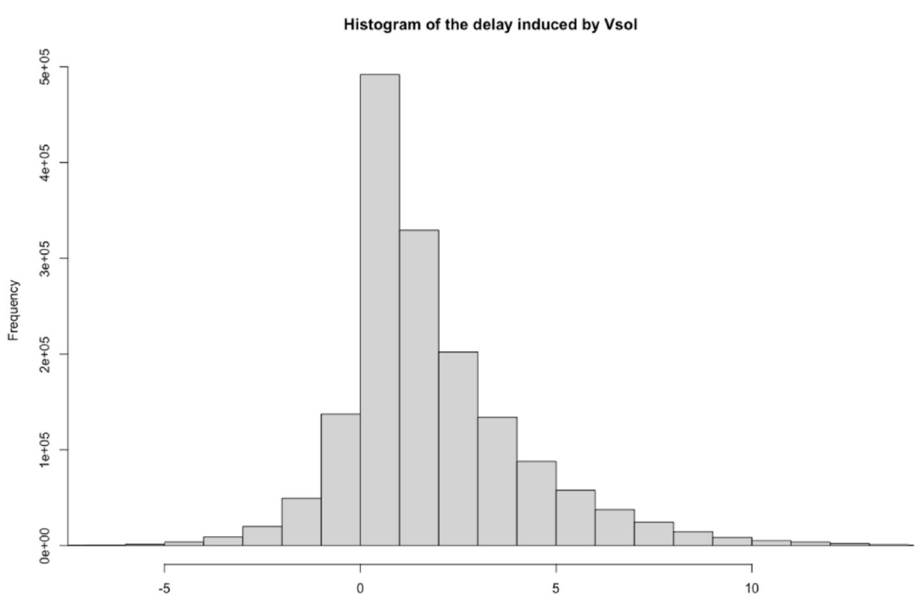


Fig. 10. Histogram of the difference of travel time for all pairs of origin/destination.

Table 2
Deciles of the increase of travel time for $p_{elim} = 0.5$.

Decile	10%	20%	30%	40%	50%	60%	70%	80%	90%
Seconds	12	30	50	70	94	125	165	222	313

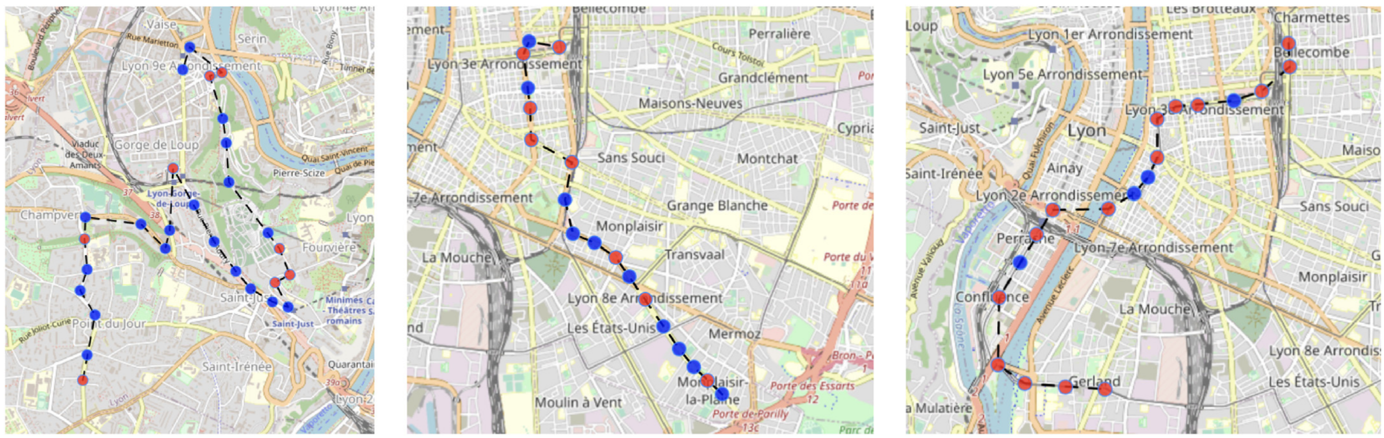


Fig. 11. Lines 90, C25 and T1 (or some sublines) with respectively 71%, 55.5% and 21% of remaining open stops.

information from loop detectors in Lyon's urban area. SymuVia has been parametrized with different origin/destination inputs until the output traffic matches the real traffic data collected with the loop detectors. So the *OD* has been built to match real cases.

The bus lines used later in this article are taken from TCL dataset (TCL, 2021).

4.2. Numerical results and analysis

Even if the inputs of the model (topology of the network and demand) are taken from real dataset, we have to choose the parameters of the problem to be realistic too. The parameters are α, $k(u)$ for all $u \in U$, and p_{elim}. For the case study, we make the following choice of parameters which is reasonable for real case study application:

1. We allow an increase of each trip by a factor $\alpha = 1.5$
2. For the users, we force that the access time does not increase more than by a factor 2, meaning that for all $u \in U$, $k(u) = 2$
3. In our case study, p_{elim} will take values between 0.1 and 0.6 (for values above 0.6, no feasible solution exist in our case, due to the value of the other parameters)

We also present results for another choice of parameters in Appendix A.

4.2.1. Optimal sub-network

Now that both inputs and parameters are set, we can create an OSNP instance described in Section 3.1.1 and solve it. In our case, we use the optimization software CPLEX (2009). We represent the solution in Fig. 9 by representing the deleted bus stops in red while the remaining ones stay in blue for different values of p_{elim}.

First of all, one interesting element to analyse is the number of deleted bus stops with regard to p_{elim}. We expect the number of deleted bus stops to be exactly equal to $\lceil p_{elim} n_t \rceil$. Table 1 shows the number of deleted nodes for different values of p_{elim}.

We notice that for $p_{elim} = 0.1$ and $p_{elim} = 0.2$, we delete more stops than what we are imposed to as the number of deleted bus stops is larger than $\lceil p_{elim} n_t \rceil$. This can be explained by the fact that for low values of p_{elim}, the optimal solution does not need that many bus stops, because the number of urban areas is limited with regard to the number of bus stops.

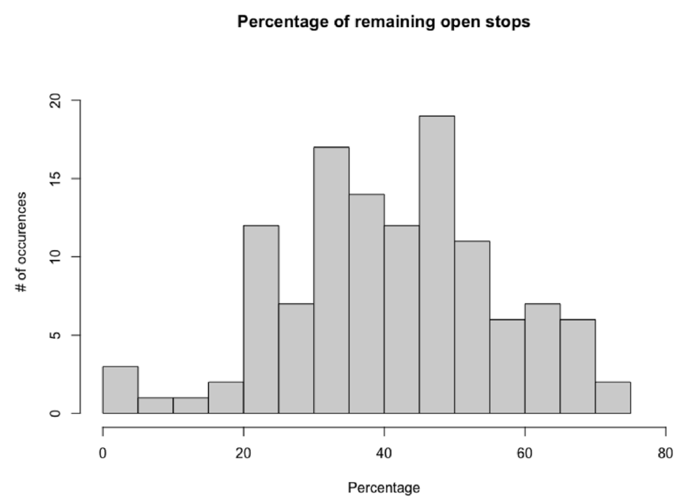


Fig. 12. Histogram of the percentage of remaining open stops for every line containing more than 10 stops for $p_{elim} = 0.5$.

Increasing the number of urban zones (n_u) would solve this side-effect. However, this would also increase the computational time needed to find a solution.

In the following, we evaluate the delay induced by the exclusion of the bus stops $V \setminus V_{sol}$. As we assumed in Assumption (5), for a pair of origin/destination $u_1, u_2 \in U$, and a solution V_{sol}, the difference of travel time is $Opt_{V_{sol}}(u_1, u_2) - Opt_V(u_1, u_2) \in \mathbb{R}$. As previously mentioned, this difference can be negative as a consequence of the fact that the shortest path between u_1 and u_2 does not traverse $acc(u_1)$ and $acc(u_2)$ in general.

We represent the histogram of the (non-zero) difference of travel time for all pairs of origin/destination in Fig. 10, for $p_{elim} = 0.5$. Similar results were obtained for other values of p_{elim}.

We note that for 86.28% of the origin/destinations, the exclusion of the bus stops induces an increase of the travel time. When it induces a decrease of the travel time, the average decrease is 67 s. When it is an increase, the average increase is 135 s. We note that the increase is rarely important, since 80% of the increase are below 222 s. More precisely, the corresponding deciles are given in Table 2.

The resolution of OSNP with real data gives us interesting

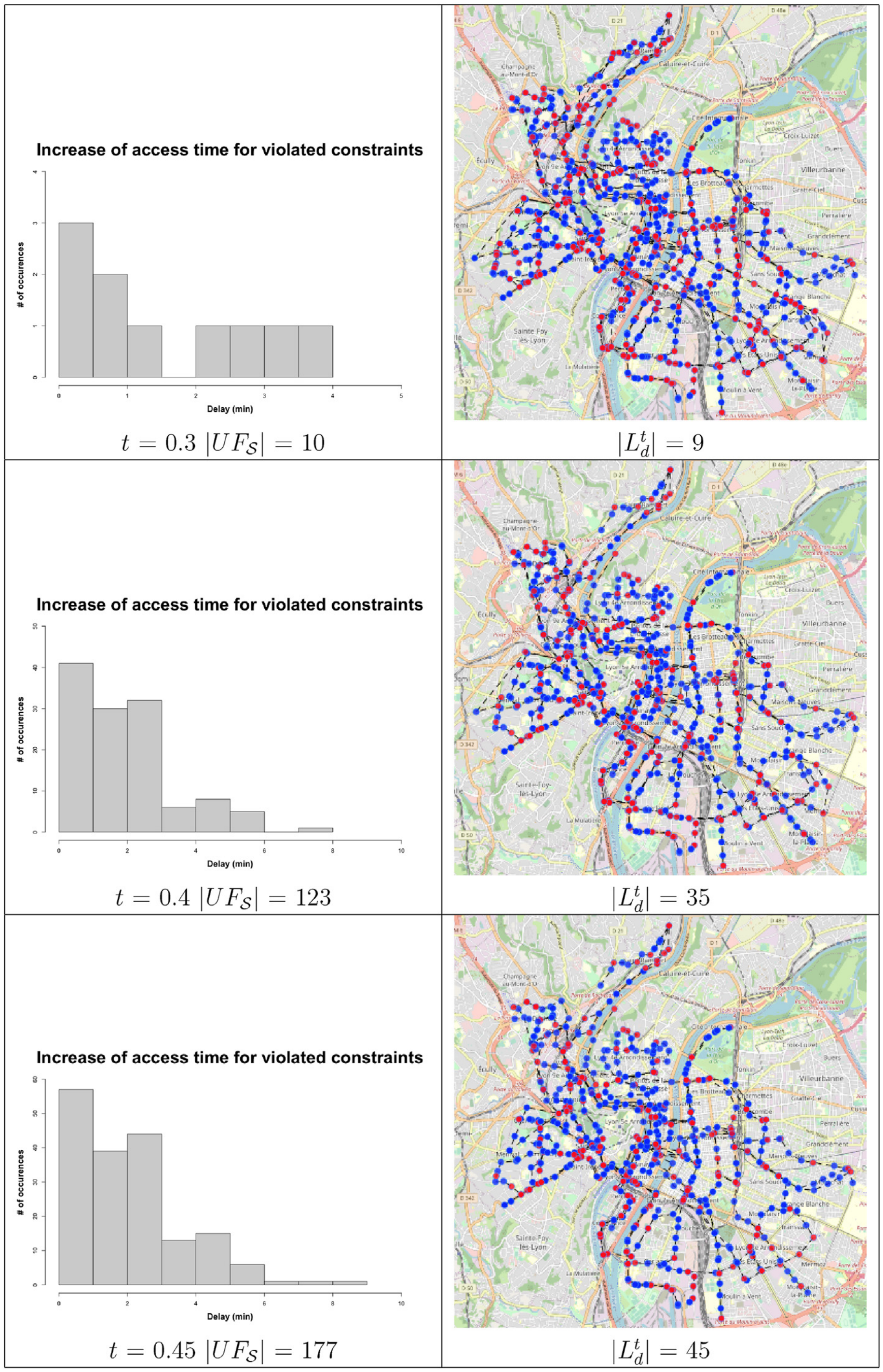


Fig. 13. $|L_d^t|$, $|UF_{\mathcal{S}}|$ and $\mathcal{H}_{uf}$ for values of t between 0.3 and 0.45 and $p_{elim} = 0.5$.

information on the bus stops we would rather keep. However, to go further in light of an operational application of the proposed approaches, we evaluate in the following multiple scenarios by using our decision tool, as described in Section 3.2.

4.2.2. Decision tool

As explained before, the consequences of the closure of some bus stops on the lines can vary significantly. As an example, we represent in Fig. 11 three lines as sequences of bus stops and we draw in red the closed bus stops while the others remain in blue. We also give the corresponding percentage of remaining open stops.

Fig. 11 clearly highlights that profiles of the lines can be very different in term of number of remaining open stops. We can plot the histogram of the percentages of remaining open stops for each line that contains more than 10 bus stops. We keep only such lines to avoid statistical aberrations: if the number of bus stops is too small, then it is more likely to have extreme values of remaining open stops. We represent such a histogram for $p_{elim} = 0.5$ in Fig. 12.

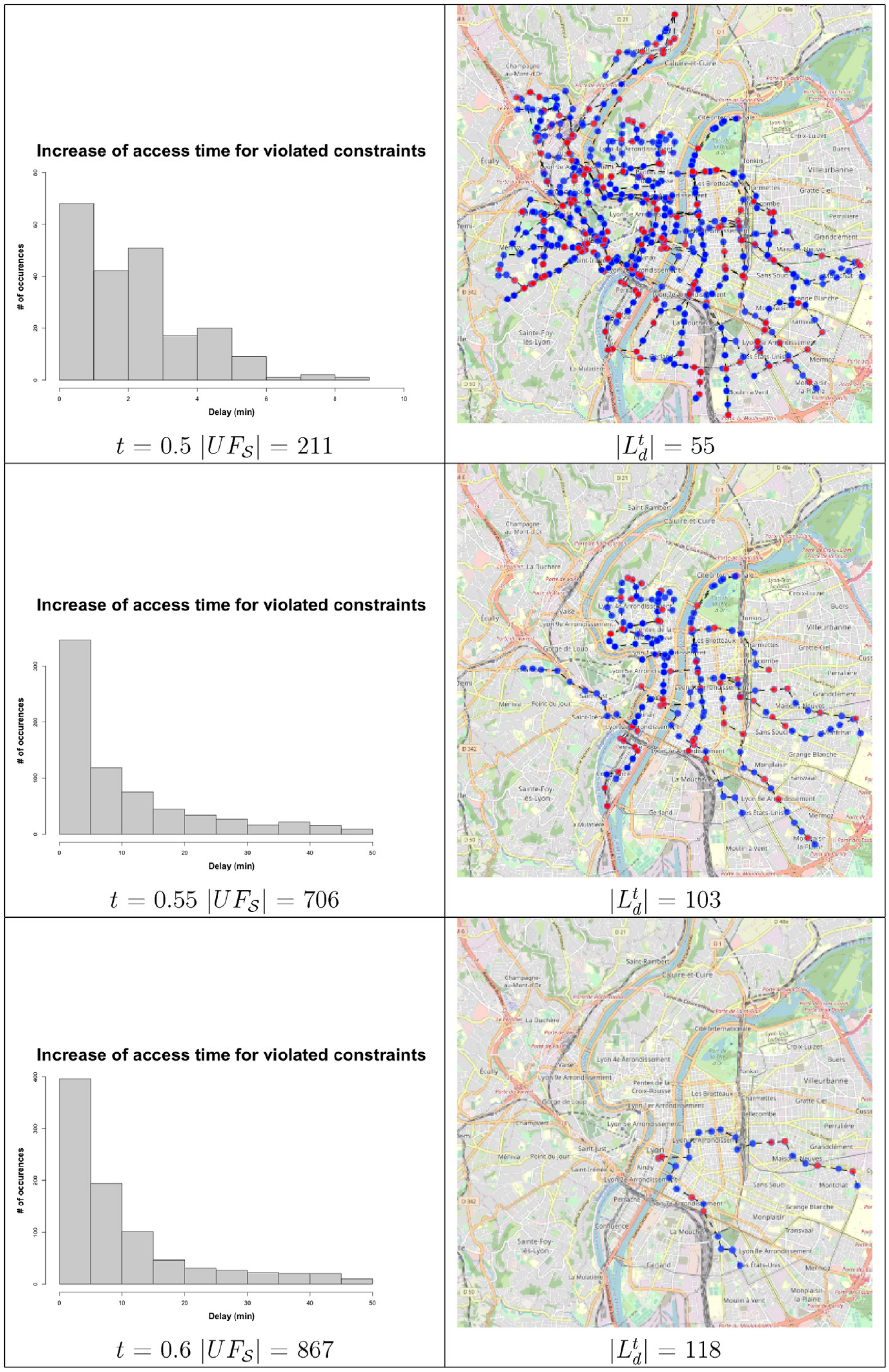


Fig. 14. $|L_d^t|$, $|UF_{\mathcal{S}}|$ and $\mathcal{H}_{uf}$ for values of t between 0.1 and 0.6 and $p_{elim} = 0.5$.

Table 3
Deciles of $\mathcal{H}_{uf}$ for $t = 0.55$.

Decile	10%	20%	30%	40%	50%	60%	70%	80%	90%
Minuts	0.92	1.93	2.75	3.79	5.17	7.16	11.60	16.90	27.03

First of all, we can see that the ideal case is not reached since most values are neither close to 0 nor close to 1 (we cannot conclude that the percentage of open stops follows a normal law either since the p-value of the Shapiro-Wilk test is $p = 0.2072$). Let us build the scenarios for different values of threshold.

We present 6 scenarios for $p_{elim} = 0.5$ and t values from 0.3 to 0.6 in Figs. 13 and 14. In this instance, we have 120 different lines containing more than 10 stops, and the number of urban zones is $n_u = 1.464$.

For each scenario we give the value of t, the number of constraints of (P) that are violated $|UF_{\mathcal{S}}|$, the number of deleted lines $|L_d^t|$, the histogram of delays induced by the deletion of the lines $\mathcal{H}_{uf}$. We also give an overview of the bus network for each scenario. We assume here that the walking speed of a user of the network is equal to 3 km/h.

First of all, we note as expected that the bigger is t, bigger is the number of deleted lines and bigger is the number of violated constraints. For the biggest values of t, we also note that the delay induced by the deletion of the lines are grouped around 0. The deciles of $\mathcal{H}_{uf}$ for $t = 0.55$ are shown in Table 3. This shows that 50% of the delays induced are approximately below 5 min, which is reasonable. Note that we consider only delays on the access time, and not the total delay including the travel time. Note also that the distance we consider is the haversine distance, and does not take into account the obstacles. Thus, the distances are lower than in the reality.

We see that as long as t increases, the network downsizes as the number of deleted lines increases. At the end, only a small part of the network still exists. For big values of t, it is clear that the assumption we made about the traveling time (Section 1) is no longer reasonable in this case. Indeed, if the network is too reduced, then the travel time cannot be considered as the same as before.

5. Conclusions and future work

In this paper we present a generic model to build a decision tool for bus network planners that provides them scenarios to adapt their network to a decreasing demand. The model is based on a MILP that finds an optimal sub-network in terms of bus stops. This sub-network describes the bus stops that we are likely to keep in order to minimize the difference of access time, the delay induced by the deletion of some bus stops and also guarantees that a minimal percentage of bus stops have been excluded. From this optimal sub-network, we have described a methodology to build scenarios that provide the network planners a subset of bus lines to keep, and a distribution of the corresponding delays induced by the network reduction.

In a second time, we have applied the generic model in Lyon's urban area with real data. After having reasonable choices of parameters, we have presented numerical results that lead to operational scenarios. The operators of networks can now choose between the scenarios according to the trade-off they prefer.

The computational running time is a big issue in our article. Indeed, we made some simplifying assumptions in order to allow the real-case resolution. A more realistic MILP could have been written (and has been), but its resolutions were in our case too long for our real-case application. However, these assumptions make the numerical results less realistic, especially when the threshold below which we delete the bus lines (t, see Section 3.2) is high. Indeed, the travel time cannot be considered as unchanged when the network downsizes. To go further, it could be interesting to dig into more complex resolution algorithms to get rid of simplifying assumptions and still get results on real instances.

Note that the choice of parameters is a real issue with regard to the computational running time too. Indeed, if we take for instance our parameter k (which describes how far the accession time is allowed to increase), then if k increases, the number of potential bus stops to look into can increase, making the corresponding instances hard to solve in reasonable time.

The guideline of this paper has been to highlight a choice of bus stops in order to choose a subset of lines at the end. However, the choice of such lines induces in the general case a set of unfeasible bus stops according to our MILP formulation. Even if we have been able to evaluate the profile of the unfeasibility, we have no guarantee *a priori* of the quality of our solution in terms of bus lines. This could be a problem in some cases. Another solution could have been to see the problem with a "line" point of view from the beginning, and to define a solution as a subset of lines to keep. However, this could increase drastically the computational complexity. Such approaches are currently studied.

In this article we provide scenarios that are different trade-offs between the number of lines that we delete on the one hand, and the delay caused by the corresponding deletion on the other hand. Once different scenarios are built, this is up to the network operators to choose according to their preferences. One other way to process would have been to compute an optimal trade-off. To do that, we should have brought in a cost function both delays and deleted bus lines. The main difficulty in this approach is the way to compare both aspects quantitatively. This would induce some economical approach of delays: how much money "costs" 1 min of delay? These approaches are very interesting and should be dug into.

Replication and data sharing

All the code needed to replicate the results are available here: https://github.com/licit-lab/optimal_subgraph.git.
The data used for the simulations are described in Section 4.1.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Results with another choice of parameters

We present results for another choice of parameters.

- $\alpha = 2$
- for all $u \in U$, $k(u) = 1.75$

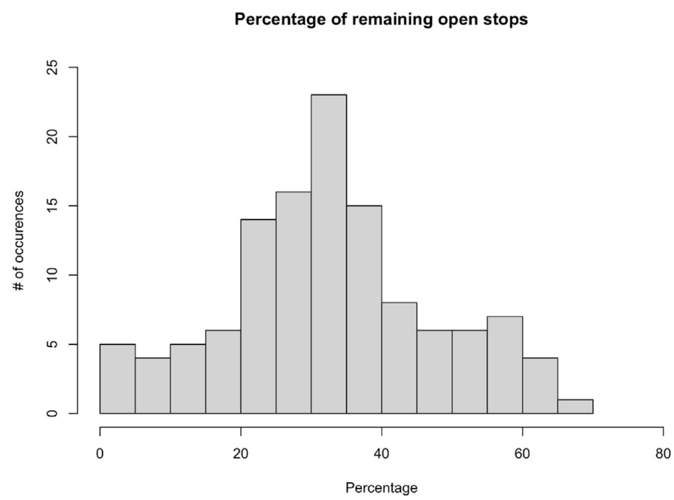

Fig. A.15. Histogram of the percentage of remaining open stops for every line containing more than 10 stops for $p_{elim} = 0.7$.

Table A.4
Number of deleted bus stops.

p_{elim}	$\lceil p_{elim} n_t \rceil$	# deleted bus stops
0.2	229	380
0.3	344	412
0.4	458	458
0.5	572	572
0.6	687	687
0.7	801	801

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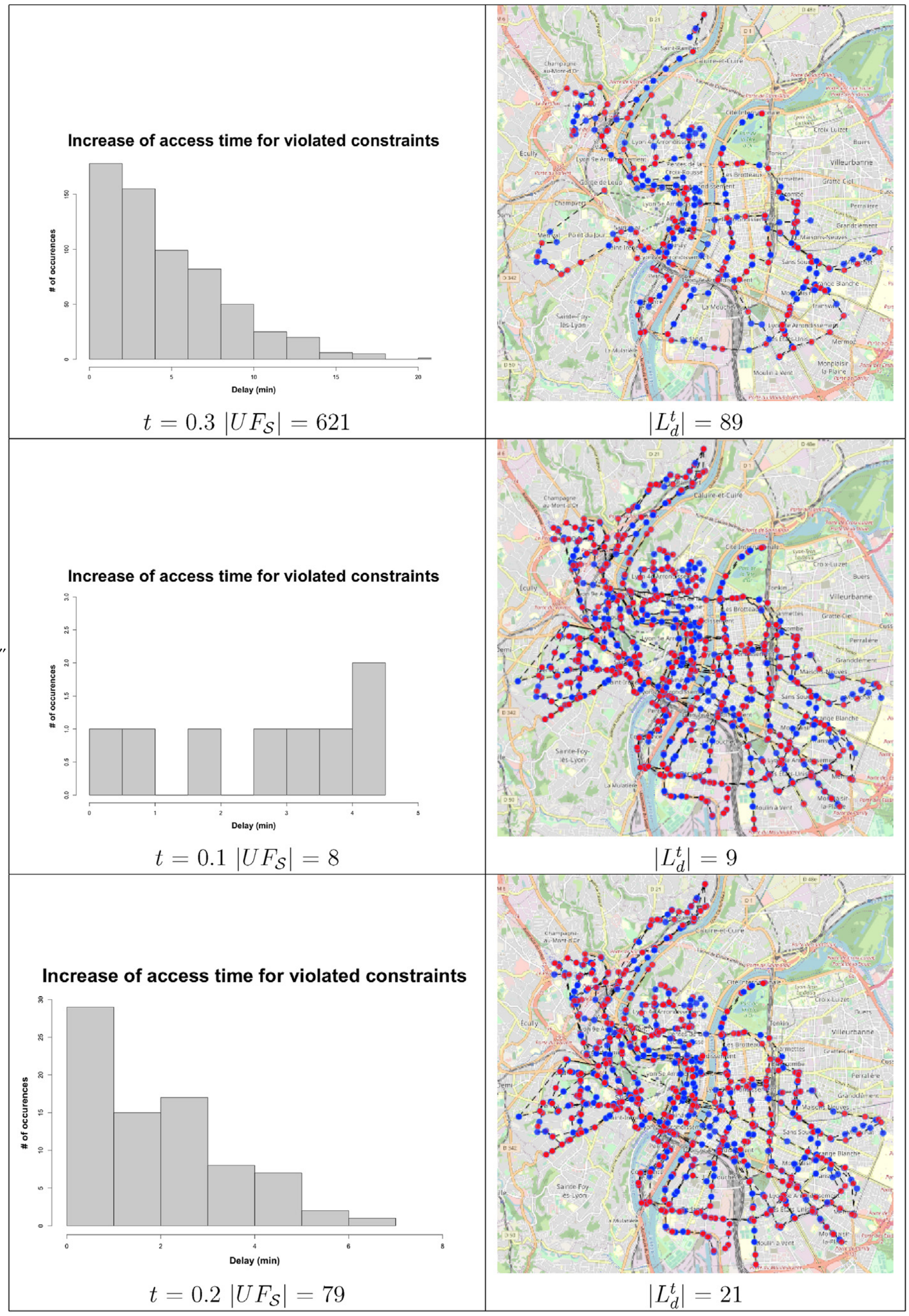

Fig A.16. $|L_d^t|$, $|UF_{\mathcal{S}}|$ and $\mathcal{H}_{uf}$ for values of t between 0.1 and 0.4 and $p_{elim} = 0.7$.

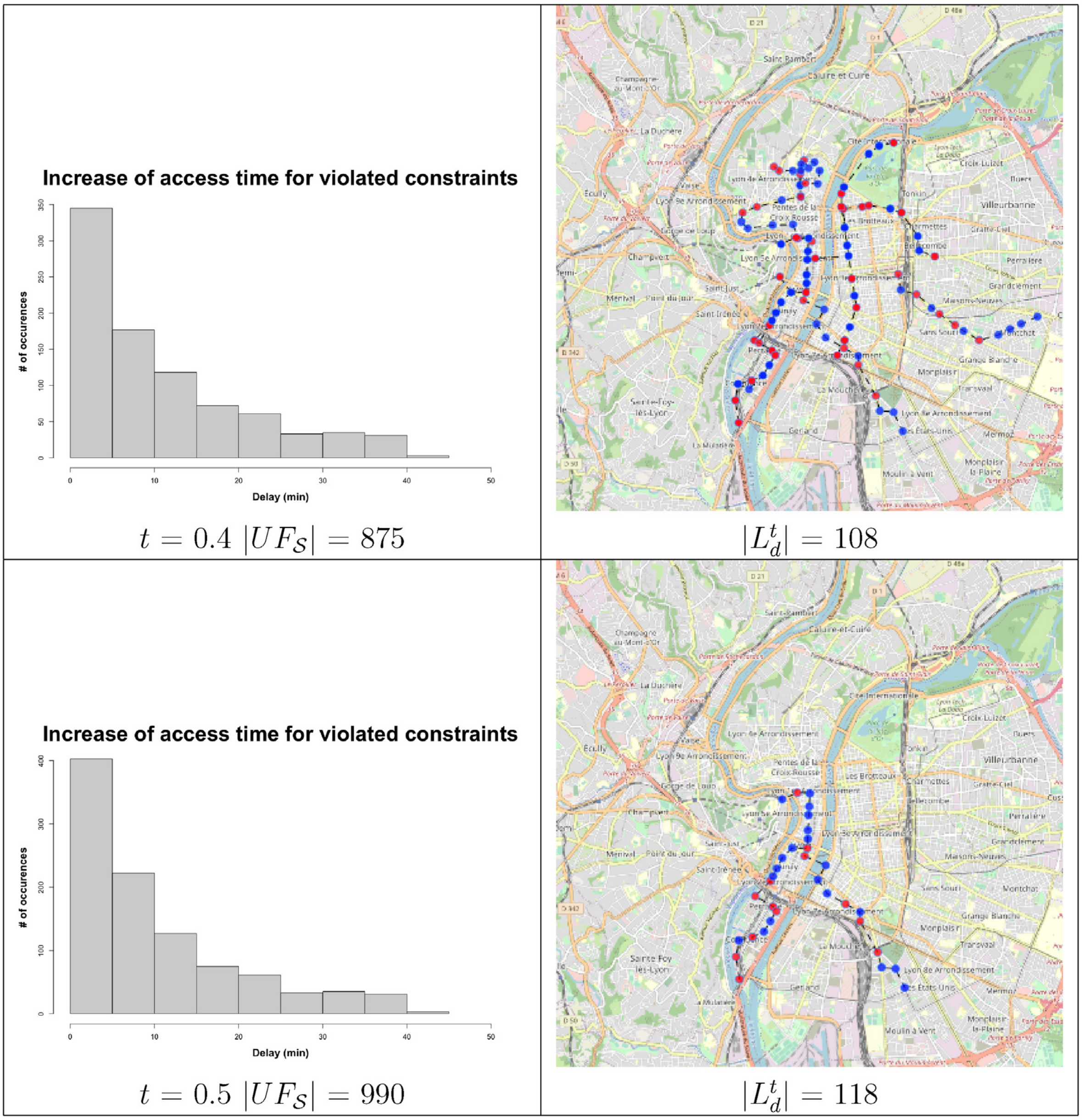

Fig. A.17. $|L_d^t|$, $|UF_{\mathcal{S}}|$ and $\mathcal{H}_{uf}$ for values of t between 0.3 and 0.5 and $p_{elim} = 0.7$.

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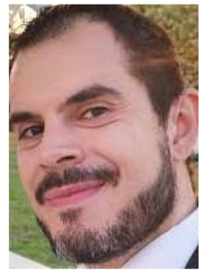

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